**Assignment 3: Linear Mixed Effect Modelling**

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PSYR 6003: Fundamentals of Applied Statistics and Research Design

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April 13, 2025

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**Results**

A linear mixed effect model was used to investigate the relationship between satisfaction with life (SWL), extraversion, and neuroticism. Three hypotheses were tested. First, it was hypothesized that extraversion will be positively associated with SWL. Second, it was hypothesized that neuroticism will be negatively associated with SWL. Finally, it was hypothesized that the effects will be similar for both level 1 (within participants over time) and level 2 (between participants).

***Data Analysis***

The data was cleaned by removing other variables from the dataset, as they were not needed for this analysis. The data was already in a ‘long’ format, so there was no need to convert it to a ‘wide format’. The columns “id”, “day”, “swl”, “tipm.E”, and “tipm.N were selected, as the assignment stated that these were the only variables needed. The variable ‘id’ was converted from a numeric to a factor variable as it is categorical. This allows us to treat the cluster variable as individual participants rather than assuming a continuous or ordinal nature. The dataset had 4252 observations in total.

Model comparison was used to determine if fixed or both fixed and random slopes should be used. For the model comparison analysis, the maximum likelihood (ML) estimate was used. This is because the models were compared using nested model comparison. While restricted maximum likelihood estimation (REML) provides more unbiased estimates and is computationally faster, ML allows for the whole model (both fixed and random effects) to be compared to a simpler model. However, when summarizing and estimating the chosen model (based on the model comparison analysis), REML was used for the reasons stated previously. When running the full model, the error that the model “failed to converge” appeared. For the purposes of this assignment, the message was ignored. However, if one wanted to address this issue, they could increase the iterations, which would increase the amount of times the ML tries different values for the parameters in the model to find the lowest log likelihood. Another way to solve this is to slightly increase tolerance, which would increase the chances of the ML finding values that are considered close enough to identical. The current package being used (“lme4”) does not allow for the covariance structure to be specified, and automatically uses an unstructured covariance structure.

***Descriptive Statistics, Distribution, and Diagnostics***

The descriptive statistics and correlations (see Table 1) suggest that the relationship between SWL and extraversion, and SWL and neuroticism will be positive and negative, respectively. Multicollinearity is not an issue as none of the correlations are large, but rather are small to moderate correlations between SWL and extraversion (*r* = .38), SWL and neuroticism (*r* = -.45), and extraversion and neuroticism (*r* = -.33). Participants had an average score of 4.18 (*SD* = 1.52) on the extraversion scale and an average score of 3.49 (*SD* = 1.54) on the neuroticism scale. SWL has an average of 4.43 (*SD* = 1.61). All variables were rated on a scale of 1 (strongly disagree) to 7 (strongly agree).

The univariate distributions showed no obvious issues with the data. SWL was slightly negatively skewed, while extraversion was slightly positively skewed. Neuroticism was normally distributed.

Statistical assumptions were tested. Given that linear mixed models can model dependence, the assumption of independence is not an issue, despite the data being repeated-measures data. The assumption of normality was met based on the histogram. If normality was a concern, a Q-Q plot can be observed to see if the data is skewed. Based on the Residual-Dependence plot, the assumption of linearity was violated, as the data has a funnel shape to it. This violation can be addressed by modelling data with polynomial terms instead of as a linear equation and doing a model comparison to see if that improves the fit of the model. Based on the S-L plot, the assumption of homoskedasticity is violated as the plot is not flat. To address this, robust standard errors can be used to solve the issues of heteroskedasticity. This method calculates the standard errors, but does not assume that they have constant variance. This will only change the standard error, not the values for the coefficients.

***Modelling the Data***

A base model was run with SWL as the outcome and no predictors to determine the Intraclass Correlation (ICC). The ICC (0.74; see Table 2) suggests that 74% of the variance is due to clustering. Design effects suggest that if a linear mixed model was not used, the analysis would be artificially increasing the sample size by 12.29 times, which would increase the Type 1 error. The ICC supports a linear mixed model approach.

An initial model was run, comparing the fixed effects model with extraversion as predictor to the model with random slopes for extraversion, showed that the model with random slopes improved the model fit (as seen in (1)). Note that, for this and following models, the intercepts are random as each participant has a different baseline. For the model with only fixed effects, the AIC = 10945.67 and the BIC = 10971.08, while the model with extraversion as a random slope had an AIC = 10865.86 and BIC = 10903.99. The AIC and BIC values were lower for the model with random slopes and the Bayes’ factor was larger than 100, suggesting decisive evidence for this model. The predicted differences between the two models was 1.254, which is a large amount given the range of the scale. This suggests that the relationship between extraversion and SWL shows variation across individual participants. The model with only fixed slopes for extraversion explained 29.7% of the variance, while the model with explained 6.07% less. Since the summary statistics show that the model with the random slope explains more variance, this decrease could be because more of the variance is accounted for individual participant differences, instead of an overall effect.

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

Next, a model comparing the random slopes model for extraversion was compared to a model that included both the random slopes for extraversion and the fixed slopes for both extraversion and neuroticism (as seen in (2)). For the model with random slopes for extraversion, the AIC = 10865.86 and the BIC = 10903.99, while the model that added the fixed effects of neuroticism had an AIC = 10498.33 and BIC = 10542.81. The AIC and BIC values were lower for the model with the added fixed effects of neuroticism and the Bayes’ factor was larger than 100, suggesting decisive evidence for this model. The predicted differences between the two models was 1.406, which is a large amount given the range of the scale. This suggests that including neuroticism in the model increases the model fit. The model without neuroticism as a fixed effect explained 38.75% of variance, while the model with explained 3.01% more.

|  |  |
| --- | --- |
|  | (2) |

Finally, a model that included both the random slopes for extraversion and the fixed slopes for both extraversion and neuroticism was compared to a model that included the fixed and random slopes for both extraversion and neuroticism (as seen in (3)). For the model with random slopes for extraversion and fixed slopes for neuroticism, the AIC = 10561.75 and the BIC = 10593.52, while the model that added the random slopes of neuroticism had an AIC = 10364.47 and BIC = 10428.02. Once again, the AIC and BIC values were lower for the model with random slopes for both predictors and the Bayes’ factor was larger than 100, suggesting decisive evidence for this model. The predicted differences between the two models was 1.035, which is a large amount given the range of the scale. This suggests that the relationship between both extraversion and neuroticism with SWL show variation across individual participants. The full model with only random slopes for both extraversion and fixed effects for neuroticism explained 15.0% of variance, while the model with random slopes for both explained 7.98% more. 17.9% of the variance was explained by the random slope of extraversion.

|  |  |
| --- | --- |
|  | (3) |

Based on the full model visualizations, it appears that there is a positive relationship between SWL and extraversion and a negative relationship between SWL and neuroticism. The average slope lines are parallel in the plot, suggesting that there are no interactions.

Overall, the full model explains 78.8% of the variance in SWL, based on the conditional R2. Based on the marginal R2, the fixed effects only explain 9.4% of variance in the model. The residual R2 indicates that 26.0% of variability in SWL is predicted day to day around the average for each participant. However, due to the negative intercept R2, none of the variability in SWL is predicted around the overall mean of SWL for all participants. When computing these estimates, a warning occurred and the package “flexplot” may have affected some of the output. Based on instructors’ suggestions, this error was ignored, as it only seemed to affect the “estimate()” command, and therefore, potentially the intercept and residual R2 output.

***Extraversion and Neuroticism in Relation to SWL***

Estimates and the model results can be found in Table 2. For the overall model, when scores on extraversion and neuroticism are zero, participants will have an SWL score of 4.51 (95%CI [4.27, 4.74], SE = 0.118). Relationships between all predictors and the outcome were statistically significant (*p* > .001). All estimates have a 95% confidence interval that does not cross zero, suggesting significance*.*

For the first hypothesis, it was hypothesized that extraversion will be positively associated with SWL. Both the full model visualization and the estimates (see Table 2) support this hypothesis (*b* = 0.161, 95%CI [0.131, 0.191], *SE* = 0.015). The estimates suggest that for every one-point increase in extraversion, scores on SWL will increase by 0.161.

For the second hypothesis, it was hypothesized that neuroticism will be negatively associated with SWL. Both the full model visualizations and the estimates (see Table 2) support this hypothesis (*b* = -0.211, 95%CI [-0.244, -0.177], *SE* = 0.017). The estimates suggest that for every one-point increase in neuroticism, scores on SWL will decrease by 0.211.

***Level 1 and Level 2 Effects***

For the third hypothesis, it was hypothesized that the effects will be similar for both level 1 (within participants over time) and level 2 (between participants). This can be determined by the model comparison outcome. As the model comparison suggested that the model with random slopes for both extraversion and neuroticism should be used over the models without, it can be determined that the effects are not similar for both level 1 and level 2. The full model allows for the slopes of both predictors to vary from participant to participant, suggesting that individual differences in scores on extraversion and neuroticism play a large role in the variability within SWL. If effects were similar across level 1 and level 2, allowing slopes to vary would not have shown a difference in model fit between the models with fixed effects and the models with both fixed and random effects. The results show that the variance explained by the model with both fixed and random effects (78.8%) explains more than the variance explained by only the fixed effects (9.4%), supporting the result that the effects are different from level 1 to level 2. It can also be seen in the variance of the random effects that the cluster variable of ‘id’ explains most of the variance (2.025) that is accounted for by the random effects, suggesting that participants differ in their baseline scores. The random slopes for extraversion (0.020) and neuroticism (0.035) are smaller, suggesting that these variables varied from participant to participant more moderately. Finally, the residual variance (0.488) indicates that there is still a moderate amount of unexplained variability after accounting for the fixed and random effects. These effects were interpreted as they are relevant to the third hypothesis.

Overall, the first and second hypothesis were supported, such that extraversion is positively associated with SWL and neuroticism is negatively associated with SWL. However, the third hypothesis was not supported, such that the effects were not similar for both level 1 (within participants over time) and level 2 (between participants). The results suggest that when the model allows slopes, as well as intercepts, to differ across participants, the effects on SWL differ compared to when the slopes are held constant for all participants. This analysis found that those who score higher on extraversion are more likely to score higher on SWL, while those higher in neuroticism are more likely to score lower on SWL. This relationship, however, varies across individuals. These results should be interpreted with caution due to the issues with linearity and homoskedasticity.

**References**

Stanley, D. (2023, June 26). apaTables. <https://dstanley4.github.io/apaTables/articles/apaTables.html>

Yakovenko, I. (2024, April 12). PSYR6003-Assignment-4. GitHub. <https://github.com/iyakoven/PSYR6003-Assignment-4>

**Table 1**

*Means, Standard Deviations, and Correlations with Confidence Intervals*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variable | *M* | *SD* | 1 | 2 |
|  |  |  |  |  |
| 1. SWL | 4.43 | 1.61 |  |  |
|  |  |  |  |  |
| 2. Extraversion | 4.18 | 1.52 | .38\*\* |  |
|  |  |  | [.35, .41] |  |
|  |  |  |  |  |
| 3. Neuroticism | 3.49 | 1.54 | -.45\*\* | -.33\*\* |
|  |  |  | [-.47, -.43] | [-.36, -.31] |
|  |  |  |  |  |

*Note.* *M* and *SD* are used to represent mean and standard deviation, respectively. Values in square brackets indicate the 95% confidence interval for each correlation. \* indicates *p* < .05. \*\* indicates *p* < .01.

**Table 2**

*Summary of Model Results With Random Effects*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Satisfaction With Life** | | | |
| *Predictors* | *Estimates* | *SE* | *CI* | *p* |
| (Intercept) | 4.506 | 0.118 | 4.274 – 4.737 | **<0.001** |
| Extraversion | 0.161 | 0.015 | 0.131 – 0.191 | **<0.001** |
| Neuroticism | -0.211 | 0.017 | -0.244 – -0.177 | **<0.001** |
| Marginal R2 / Conditional R2 | 0.094 / 0.788 |  |  |  |

*Note.* *SE* represents the standard error, while *CI* represents the confidence interval. *p*-values were included as linear mixed models account for non-independence, and therefore do not inflate the *p*-value, making it more reliable.